RESEARCH STATEMENT

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I am a geometer; particularly, I like studying Teuchmuller spaces - (higher) higher ones, and those corresponding to infinite-type surfaces. To me, these objects often represent some kind of *geometric structure* on a manifold. I like taking a geometric perspective on problems, being at my happiest when I can draw accurate and illuminating pictures.

Since Klein's Erlangen program, a fruitful avenue of research has been studying the geometry of manifolds and orbifolds via realizing their fundamental groups as subgroups of Lie groups, and vice versa. For example, all discrete subgroups of $\mathrm{PSL}_2(\mathbb{R})$ arise from, and determine, a surface (or orbifold) endowed with a constant negative curvature metric; all discrete subgroups of $\mathrm{PSL}_2(\mathbb{R})$ act properly discontinuously on the hyperbolic plane, and thus have nice quotients. The classical Teichmüller space parametrizes these structures. My main interests are in what happens when we take a more complicated group in place of $\mathrm{PSL}_2(\mathbb{R})$, such as, for example, the group of affine transformations, and when we take a more complicated manifold instead of a surface, for instance a three-manifold or an infinite-type surface.

The study of these objects makes use of and further informs areas of math such as geometric structures on manifolds, projective and (pseudo-)Riemannian geometry, Lie theory, Higgs bundles, dynamics, low-dimensional topology, algebraic topology, and algebraic geometry, as well as geometric group theory, the study of surfaces, their mapping class groups, and their associated objects.

After a brief informal introduction to the context of my field, I will cover my past and current research in more depth and rigorous mathematical detail in subsequent sections:

- In Section 1, I recall past projects:
  - My paper [ŽK] introduces *higher strip deformations*, which allow us to construct many affine actions with different behaviors. For example, I use the construction to show:

    Theorem 1. *Every Schottky free group in* $\mathrm{SO}(2n, 2n-1)$ *admits an open family of* $\mathbb{R}^{4n-1}$*-valued cocycles giving rise to proper affine actions on* $\mathbb{R}^{4n-1}$.

  - In joint work with J.-P. Burelle [BŽK], we study the limiting behavior of higher strip deformations, obtaining:

    Theorem 2. *Every positive free group in* $\mathrm{SO}(2n, 2n-1)$ *admits proper affine deformations into* $\mathrm{SO}(2n, 2n-1) \ltimes \mathbb{R}^{4n-1}$. *Some of these actions admit fundamental domains bounded by finitely many piecewise-linear surfaces called* crooked planes.

- In Section 2, I discuss current projects involving studying affine manifolds.
  - With C. Monroe and T. Weisman, we are studying geometric constructions on three-manifolds giving rise to affine actions.
  - With C. Monroe and R. Canary we are seeking for obstructions to the existance of proper affine actions of various groups in certain dimensions, with a focus on surface and three-manifold groups.
  - Jointly with Burelle, we are seeking to answer natural questions about our construction from [BŽK].
- In Section 3, I go over projects further afield from affine actions, but mostly dealing with free groups.

  - With S. Ghosh, we are seeking to adapt methods from [**?**] to the setting of curved manifolds. Our interest is in constructing and classifying proper actions of free groups on domains in pseudo-hyperbolic space $\mathbb{H}^{n,n-1}$.
  - Jointly with D. Douglas, Z. Greenberg, C. Kineider, and E. Rogozinnikov, we are building a framework for studying the $\mathrm{Sp}(2n)$ character variety by thinking of $\mathrm{Sp}(2n)$ as SL_2 with matrix entries. This work seeks to answer a question of Wienhart.
- In Section 4, I talk about a project with G. Domat, C. Monroe, and T. Weisman on Teichmüller spaces of infinite-type surfaces via the perspective of representations of the infinitely generated free group.
- In Section 5, I briefly outline four of the projects I hope to tackle some day, but that I am not currently putting active time into. These would benefit from collaborations with representation theorists, dynamicists, differential geometers, low-dimensional topologists, and geometric group theory experts.
- In Section 6, I briefly cover some of the undergraduate research projects I've supervised.

My past work and several of my current projects are about *affine manifolds*. These are smooth manifolds with charts valued in real affine space $\mathbb{A}^n$ modeled on $\mathbb{R}^n$, where the transition maps are in the affine group $\mathrm{Aff}(\mathbb{A}^n) = \mathrm{GL}_n(\mathbb{R}) \ltimes \mathbb{R}^n$. When the affine manifold M is complete, that is, when we can write M as a quotient of $\mathbb{A}^n$ by a discrete group $\Gamma < \mathrm{Aff}(n)$, we can easily pass between studying M and studying its fundamental group Γ. Unlike in the case of hyperbolic surfaces and $\mathrm{PSL}_2(\mathbb{R})$, the group $\mathrm{Aff}(n)$ does not preserve a metric on $\mathbb{A}^n$, and not every discrete subgroup will act properly discontinuously on $\mathbb{A}^n$. Thus it is natural to ask, and it is the driving goal of much of my research:

Question 3. Which groups can arise as fundamental groups of complete affine manifolds? Is there some systematic way of classifying all complete affine structures on a manifold?

The Auslander conjecture [Aus64], a long-standing open problem in the field, posits that only polycyclic groups can be fundamental groups of compact complete affine manifolds. This is in analogy with Bieberbach's theorem from Euclidean geometry - virtually, only free abelian groups appear as fundamental groups of Euclidean manifolds, even when we do not require compactness.

Margulis was first to discover the necessity of of the compactness condition. There are three-dimensional noncompact complete affine manifolds with free fundamental groups, now called Margulis space-times. They have been fully classified by Danciger-Guéritaud-Kassel [DGK16]. Their approach relies on the beautiful coincidental isomorphism between the linear and adjoint representations of $\mathrm{SO}(2,1)$, naturally suggesting two paths forward. For a Lie group $G < \mathrm{GL}_n(\mathbb{R})$, we can study groups in $G \ltimes \mathfrak{g} < \mathrm{Aff}(\mathfrak{g})$, acting properly discontinuously on the Lie algebra $\mathfrak{g}$ of G, or we can look for groups in $G \ltimes \mathbb{R}^n < \mathrm{Aff}(\mathbb{A}^n)$, acting properly discontinuously on $\mathbb{A}^n$.

My past work helps elucidate the second generalization, introducing flexible and powerful constructions for creating proper affine actions of free groups in every $\mathbb{R}^{4n-1}$. Two of my current projects are focused on the first generalization, harnessing the strength of the geometric connection between a Lie group and its Lie algebra.

1. Past results

1.1. **Proper affine deformations of irreducible free groups.** In my paper [ŽK] I introduce higher strip deformations, a tool for constructing proper affine actions of a free group on any $\mathbb{A}^{4n-1}$, with the linear part of the group lying in the image of the irreducible representation $\sigma_{4n-1} : \mathrm{PSL}_2(\mathbb{R}) \to \mathrm{SL}_{4n-1}(\mathbb{R})$. The irreducibility condition gives us strong connections to the hyperbolic geometry of noncompact surfaces. This allows us to explicitly define cocycles in terms of data on the hyperbolic surface in question.

Let S be a noncompact surface. Equipping S with a complete infinite-volume hyperbolic metric realizes the fundamental group of S as a discrete free group $\Gamma < \mathrm{PSL}_2(\mathbb{R})$ with no parabolic elements. Then $\sigma_{4n-1}(\Gamma)$ is a discrete subgroup of $\mathrm{SO}(2n, 2n-1)$, and all its elements are diagonalizable. A $\sigma_{4n-1}(\Gamma)$-cocycle $u : \Gamma \to \mathbb{R}^{4n-1}$ defines a homomorphism $\rho_u : \Gamma \to \mathrm{Aff}(\mathbb{A}^{4n-1})$ via $\rho_u(\gamma) = (\sigma_{4n-1}(\gamma), u(\gamma))$.

Fix an arc system on S and decorate each arc with a point and a geodesic going through it. Given any $\gamma \in \Gamma$, we can define a cocycle $u(\gamma)$ as a sum of certain vectors corresponding to each intersection between γ and the arcs. A cocycle obtained in such a way is called a *higher strip deformation*. Its explicit form allows us to obtain a constructive proof of

Theorem 4 ([ŽK])**.** *Let $S = \Gamma\backslash\mathbb{H}^2$ be a noncompact convex cocompact surface and $\sigma_{4n-1} : \mathrm{PSL}_2\,\mathbb{R} \to \mathrm{SO}(2n, 2n-1)$ the irreducible representation. Then $\sigma_{4n-1}(\Gamma)$ admits an open cone of cocycles determining proper affine actions on $\mathbb{A}^{4n-1}$.*

When Γ has some extra dynamical properties, a similar result was shown by Smilga in [Smi14]. In terms of [ŽK], some of the examples from Smilga's work can be interpreted as higher strip deformations with a particular choice of decoration of the arc system. For those choices, we recover Smilga's dynamical conditions and show their necessity.

One of the main tools used in the proof of Theorem 4 is the Margulis invariant, a class function $\alpha_u : \Gamma \to \mathbb{R}$ first introduced by Margulis in [Mar84, Mar83]. It records the signed translation length of an element of Γ_u along its invariant axis. As shown in [GLM04, GT22], the strict positivity of this invariant is sufficient to determine that the action of Γ_u on affine space is proper. For higher strip deformations, I obtain an explicit formula for the Margulis invariant. It is a rational function in the cross-ratio of the endpoints of γ and the geodesics associated to each arc.

These techniques also apply to virtually free groups (or orbifold surfaces). For example, I apply these techniques to the 7-dimensional irreducible representation of $\mathrm{PSL}_2(\mathbb{Z})$, showing

Theorem 5. *Let $\Gamma \cong C_2 \star C_3$ be a convex cocompact deformation of $\mathrm{PSL}_2\,\mathbb{Z}$ in $\mathrm{PSL}_2\,\mathbb{R}$. Then every non-trivial affine deformation of $\sigma_7(\Gamma)$ determines a proper affine action on $\mathbb{R}^7$.*

Note that in this section, we needed the assumption of Γ not containing any parabolic elements. I am aware of work in progress by Kassel and Smilga on the Margulis invariant in the presence of parabolics, which should allow all the results of this section to hold in that case as well.

1.2. **Proper affine deformations of positive free groups.** In joint work with J.-P. Burelle, we adapt higher strip deformations to the more general case of the linear part being any positive Anosov free group. We also construct fundamental domains for a large family of these examples, bounded by piece-wise linear walls, called crooked planes. In particular, this shows the *tameness* of some of these higher-dimensional Margulis spacetimes - they are all diffeomorphic to interiors of compact manifolds with boundary.

If we realize a free group Γ as a fundamental group of a noncompact surface S, that equips the boundary of Γ with a cyclic order. We can then talk about representations $\rho : \Gamma \to \mathrm{SO}(2n, 2n-1)$ that preserve said order; for a representation that is nice enough (say, Anosov), there is a ρ-equivariant map $\xi : \partial\Gamma \to \mathrm{Flag}(\mathbb{R}^{2n,2n-1})$, and a notion of positivity in the flag space. One way to think about this positivity is that it transfers some kind of dynamics and combinatorics of Γ to its image.

If we cut up S into a polygon using arcs, we can extend ξ to the endpoints of the arcs as well, preserving the positivity structure. This allows us to transfer the combinatorics of the fundamental domain over to the flag space. Using this data, we define higher crooked planes, inspired by Drumm's crooked planes used in the study of three-dimensional spacetimes [DG99, Dru92] and Burelle-Treib's work on their generalizations to projective spaces [BT22]. Each crooked plane is a piecewise linear

hypersurface in $\mathbb{R}^{4n-1}$, separating $\mathbb{R}^{4n-1}$ into two pieces, each of which is a union of half the orthants corresponding to some basis.

Each crooked plane admits a two-dimensional distinguished set of vectors called the stem quadrant, translating by which makes the crooked plane disjoint with itself. We decorate each arc on S with a vector in that arc's stem quadrant. We then construct a $\rho(\Gamma)$-cocycle, called a *generalized infinitesimal strip deformation*, similarly to how we construct it in [ŽK]. The positivity of ξ allows us to make estimates on the Margulis invariant, and conclude that the constructed cocycle determines a proper affine action. To summarize,

Theorem 6. *Let $\rho : \Gamma \to \mathrm{SO}(2n, 2n-1)$ be a positive Anosov representation of a convex cocompact free group $\Gamma < \mathrm{PSL}_2(\mathbb{R})$ and let $\mathcal{A}$ be a filling system of arcs on $\mathbb{H}^2/\Gamma$. Let $\xi : \Lambda_\Gamma \cup \Lambda_\mathcal{A} \to \mathrm{Flag}^+(\mathbb{R}^{2n,2n-1})$ be a positive equivariant map. Let $u : \Gamma \to \mathbb{R}^{2n,2n-1}$ be a generalized infinitesimal strip deformation associated to $\mathcal{A}$. Then $(\rho, u)(\Gamma)$ acts properly discontinuously on $\mathbb{R}^{2n,2n-1}$.*

When the arc system used is "big enough," the disjointness properties of crooked planes take us even further. When appropriately translated, they determine a fundamental domain for $(\rho, u)(\Gamma)$:

Theorem 7. *Let $\rho : \Gamma \to \mathrm{SO}(2n, 2n-1)$ be a positive Anosov representation of a free group $\Gamma < \mathrm{PSL}_2(\mathbb{R})$ and let $\mathcal{A}$ be an overfilling system of arcs on $\mathbb{H}^2/\Gamma$. Let $u : \Gamma \to \mathbb{R}^{2n,2n-1}$ be a generalized infinitesimal strip deformation obtained from this data. Then $\mathcal{D}$ is a fundamental domain for the action of (Γ, u) on $\mathbb{R}^{2n,2n-1}$.*

Remark 8. Note that throughout Section 1, we needed the assumption of Γ not containing any parabolic elements. I am aware of work in progress by Kassel and Smilga on the Margulis invariant in the presence of parabolics, which should allow all the results of this section to hold in that case as well.

Remark 9. Both the constructions in [ŽK] and [BŽK] recover some of the results of [DGK16] that hold in three dimensions. [BŽK] also recovers some of the results of [Smi14]. Our results of Theorem 6 are stronger, covering a bigger variety of possible linear parts. However, it is not immediately clear what the relationship between Theorem 7 and Smilga's tennis ball domains is.

2. Current projects on affine manifolds

This work naturally suggests some further directions to explore, and introduces techniques useful to tackling other problems. I am currently working on some of those with collaborators, and plan on developing further ones in the future.

2.1. Immediate further questions from [BŽK].

Our techniques for constructing fundamental domains from Theorem 7 only work when we have enough arcs. Burelle and I are exploring if our approach can be adapted to the other cases as well, and if not, can we detect the "good" cases with other tools and recognize them in the space of all "proper deformations" with given linear part.

On that note, in work discussed in Section 1 we do not claim we have access to the whole space of the possible proper deformations. By taking linear combinations of (infinitesimal) higher strip deformations, we do get an open set of proper directions, and the techniques do allow us to approach at least some pieces of the boundary. The question of if this accounts for all of them remains open.

We are also on the hunt for a good parametrization of the set of (all) proper cocycles with a given linear part. There is some overcounting with our current techniques; in a sense, there are too many ways of choosing how to extend the boundary map ξ. In three dimensions, [DGK16] normalize data on an arc system to avoid overcounting, and we are trying to find a good analogue in our setting. We have had some promising discussions about using a kind of "tripod" construction in place of an arc construction for $n = 2$, though it seems it would be difficult to generalize.

2.2. **Proper affine actions of other groups.** Another direction is to not limit ourselves to free groups, but consider fundamental groups of more complicated manifolds as well. There are two reasons we might expect to have an easier time finding non-free groups in the adjoint representation as opposed to the linear one. First, the adjoint representation is usually bigger, giving us more room to work in. Second, the adjoint representation retains more geometric information about the starting manifold. We can often realize a Lie algebra valued cocycle as a non-infinitesimal deformation of a geometric structure on the manifold. I have two ongoing projects in this direction, one joint with C. Monroe and T. Weisman, and one with R. Canary and C. Monroe.

With Monroe and Weisman we are focusing on the three-manifold case. Danciger-Guéritaud-Kassel show as a byproduct of their work on affine representations of right-angled Coxeter groups [DGK20] that hyperbolic three-manifold groups admit proper affine actions. They are often on high-dimensional spaces, and the result is using strong but non-constructive theorems from three-manifold theory. We are exploring various kinds of (branched) bending and other similar deformations into various Lie groups containing $\mathrm{PSL}_2(\mathbb{C})$ to find a more geometric reason for the existence of affine actions of three-manifold groups, and to bring the dimension of the affine space down when possible.

This work will involve studying the non-infinitesimal deformations of three-manifolds as well. This is of independent interest to my personal main goal of the project, and might take us in unexpected directions.

Our plan of attack with Canary and Monroe is to look more generally into which hyperbolic groups can admit proper affine actions. We've shown that their linear parts have to be discrete and (virtually) faithful. We currently conjecture that among hyperbolic groups, only (virtually) free ones can act properly on $\mathbb{A}^k$ for $k \leq 5$, and that no closed hyperbolic three-manifold groups act properly on $\mathbb{R}^6$.

In dimensions 4 and 5, the space essentially has to split into the well-understood three-dimensional factor and a factor with a nearly trivial action. In dimension 6, Mostow rigidity obstructs any cocycles coming from a deformation of a hyperbolic structure on the three-manifold, and we conjecture that any cocycles with a "worse" linear part will fail properness. Ideas from [AMS02, AMS12] are useful in the study of this question, though we would be more satisfied with a more geometric and less Lie-theoretic proof.

It is known from both [DGK20] and [BCS26] that surface groups *do* act properly on $\mathbb{A}^6$, but not with Hitchin linear part [DZ19]. Another interest of our project is to classify which representations of surface groups in $\mathrm{SL}_6(\mathbb{R})$ can appear as linear parts of proper affine actions.

3. Other Lie groups

3.1. **Pseudo-hyperbolic space.** Together with S. Ghosh, we are researching analogous questions to those in Section 1 for proper actions on pseudo-hyperbolic space $\mathbb{H}^{n,n-1}$. It seems likely we can get a similar result to my results with Burelle in that setting as well. The main problem we need to work around is that the positivity structures of $\mathrm{SO}(n, n-1)$ and $\mathrm{SO}(n, n)$ do not play nicely with each other. This makes some of our current work a bit ad-hoc, as the differences are forcefully reconciled. We hope to find a more unified way to look at both cases at once.

This would also lead to understanding what "non-infinitesimal" data affine deformations in the sense of Section 1 are coming from. We hope to show that they are infinitesimal versions of actions on $\mathbb{H}^{2n,2n-1}$ via a geometric transition argument. Ideally, proper affine actions of free groups in $\mathrm{SO}(n, n-1) \ltimes \mathbb{R}^{2n-1}$ would arise as infinitesimal versions of nice deformations of a free group in $\mathrm{SO}(n, n)$.

3.2. **The symplectic group.** In a project joint with D. Douglas, Z. Greenberg, C. Kineider, and E. Rogozinnikov, we are exploring the $\mathrm{Sp}(2n)$ (decorated) character variety for a punctured surface

group, from the lens of viewing $\mathrm{Sp}(2n)$ as SL_2 over the ring of $n \times n$ matrices, building a "non-commutative SL_2 theory". This would expand upon [ABR+21, GKW25] and answer a question of Wienhard.

In the classical $n = 1$ case, lambda-lengths associated to a triangulation parametrize the decorated Teichmuller space, with a nice cluster-like relationship when we flip an edge of a triangulation. Tropicalizing the lambda-length coordinates gives rise to parametrizations of the boundary of Teichmuller space. Fock-Goncharov duality allows us to pass between decorated and undecorated Teichmuller spaces. We've discovered that many of these results rely fundamentally on positivity properties of $\mathbb{R}$ or $\mathbb{C}$, which we no longer have access to when working with matrices. Perhaps the most natural analog of positivity in the realm of matrices is being positive semi-definite. So far it seems like too many elements are not comparable to yield a full and rich theory, but we may be able to give a description of some slices of the character variety, while also trying other notions of positivity.

4. Hyperbolic structures on big surfaces

Perhaps less related to the rest of my research program is my interest in big surfaces and big mapping class groups. When we think about questions of Teichmüller spaces for those, things become much less clear and the behavior is unpredictable and strange.

A big surface is a surface with an infinitely generated fundamental group, for example, we can connect sum infinitely many tori. We can still talk about equipping such a surface with a hyperbolic structure. However, how exactly we can think about deforming such a structure becomes more murky. A few approaches are presented in [Tap23, CC24, BC24].

In a project with G. Domat, C. Monroe, and T. Weisman, we are approaching thinking about the Teichmüller space of a big surface via studying the representations of the free group on countably many generators into $\mathrm{PSL}_2(\mathbb{R})$. The question of which topology to put on such a space immediately becomes pertinent. The product topology, for example, exhibits some pathological behavior different from the finitely generated case; the subsets of representations corresponding to a fixed topological type are neither closed nor open:

Theorem 10 (in progress)**.** *Let $F_\infty =< a_1, a_2, \dots >$ be an infinitely generated free group. Suppose $\rho : F_\infty \to \mathrm{PSL}_2(\mathbb{R})$ is a discrete and faithful representation so that $\mathbb{H}^2/\rho(F_\infty)$ is an infinite-type surface S. Then for any other infinite-type surface S' with at least as many punctures as S and for every $N \in \mathbb{N}$, there exists a discrete and faithful representation $\rho' : F_\infty \to \mathrm{PSL}_2(\mathbb{R})$ so that $\mathbb{H}^2/\rho'(F_\infty) = S'$ and $\rho(a_i) = \rho'(a_i)$ for all $i < N$.*

5. Further-flung future directions

5.1. **Connections to the Auslander conjecture.** The Auslander conjecture, posing that fundamental groups of closed complete affine manifolds are virtually solvable, is known for manifolds of dimension at most 6. The next natural case to look at is manifolds with linear holonomy in the 7-dimensional representation of G_2. Having a good understanding of different ways for free groups with G_2 linear part to act on $\mathbb{A}^7$ is a first step to untangling the Auslander conjecture in this case. Higher strip deformations give a concrete family of examples to work with. The case of higher strip deformations for $\mathrm{PSL}_2\,\mathbb{Z}$ in dimension 7 (see Theorem 5) suggests that there is some special behavior or extra symmetry of strip deformations in dimension 7 not present in higher dimensions which might restrict possible ways for a free group with linear part in $\mathrm{SO}(4,3)$ or G_2 to act properly on affine 7-space, suggesting there is not "enough room" for fitting together enough free groups to make an affine crystallographic group in dimension 7.

5.2. **Shape of the cone of proper actions.** Related to the topics discussed in Subsection 2.1, particularly on the parametrization of the cone of proper actions with a given linear part, is the following question:

Question 11. Let $\Gamma = \pi_1(S)$ be a quasi-Fuchsian free group in $\mathrm{SO}(2n, 2n-1)$. Is there a natural proper subset of geodesic currents P_n on S such that $\alpha_u(\gamma) > 0$ for each $\gamma \in P_n$ means that u determines a proper affine action on $\mathbb{A}^{2n,2n-1}$?

For actions on three-dimensional affine space, the set P_1 is the set of measured laminations. For $n = 2$ and the three-holed sphere, we need more data than just the measured laminations (or, in this case, the simple closed curves).

5.3. **Fock-Goncharov coordinates.** Fock-Goncharov coordinates [FG03, BD17] give coordinates on some higher Teichmüller spaces. I would like to better understand their relationship with affine actions and Margulis invariants. The relationship between the G-character variety and groups in $G \ltimes \mathfrak{g}$ should be fairly accessible. I am more interested in any kind of relationship between the coordinates and groups in $G \ltimes V$, where V is some other (e.g. the standard linear) representation of G.

5.4. **Big surfaces.** The Margulis invariant extends to geodesic currents, and the proof that the positivity of the Margulis invariant insures a proper action relies on the compactness of the space of geodesic currents. For infinite-type surfaces, this space is very big and not compact. I would like to discover more about it, and see which results using geodesic currents carry over to the infinite-type surfaces.

6. Undergraduate research

I have supervised several introductory research project through programs like Geometry Lab at the University of Texas at Austin and LoG(M) at the University of Michigan. Often, a goal of these projects is to create a visualization tool or provide a computation related to a research topic.

At Austin, F. Stecker and I supervised a team of undergraduates (S. Bruce, M. Nakos, J. Teague, A. To) that created a widget to visualize the Benzecri cocompactness theorem.

Last semester, I supervised a team of four undergraduates (A. Marx, A. Im, E. Tong, Y. Zhang) on a project about constructing and visualizing domains of discontinuity and fundamental domains for proper actions of free groups on affine three-space. In [KL24], domains of discontinuity for proper affine actions are described fairly abstractly; the goal of the project was to interpret and visualize the results of the paper in three dimensions. We were successful in generating some convincing visuals. I am currently working with Tong on proving our candidate fundamental domains in three dimensions indeed work, and on reproving the results of [KL24] in three dimensions using more elementary means.

I am currently supervising a team of undergraduates (B. Jui, P. Li, S. Shady, K. Vu), with help from graduate student G. Shemy, working on positivity in $\mathrm{SO}(2,2)$. The eventual goal is to understand the positivity structure on the boundary of AdS^3 and to create visualization tools for it. We are currently developing what I hope to be a fairly general tool for drawing pictures in $\mathbb{R}\mathrm{P}^3$.

I am also supervising an independent study/research project by undergraduate M. Zhang in computational Euclidean geometry; he is looking to prove optimization theorems for a certain notion of "thickness" of n-gons in Euclidean space.

Some of my proposed future research directions have easier to approach cases or computational approaches that could help support or dispel various conjectures along the way. I can adapt those to undergraduate or masters research projects.

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